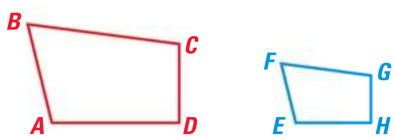
6.3 Use Similar Polygons

Before	You used proportions to solve geometry problems.
Now	You will use proportions to identify similar polygons.
Why?	So you can solve science problems, as in Ex. 34.

Key Vocabulary
- **similar polygons**
- **scale factor**

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, *ABCD* is similar to *EFGH*. You can write "*ABCD* is similar to *EFGH*" as *ABCD* ~ *EFGH*. Notice in the similarity statement that the corresponding vertices are listed in the same order.



ABCD ~ *EFGH*

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Ratios of corresponding sides

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

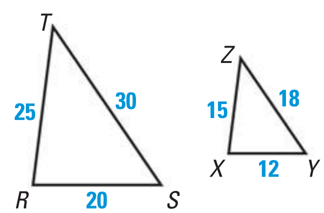
EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



READ VOCABULARY
In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

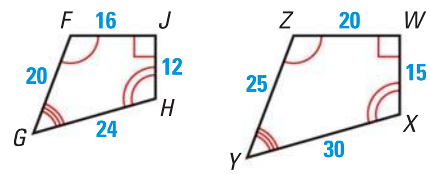
GUIDED PRACTICE for Example 1

1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

SCALE FACTOR If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



Solution

STEP 1 **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles W and J are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

STEP 2 **Show** that corresponding side lengths are proportional.

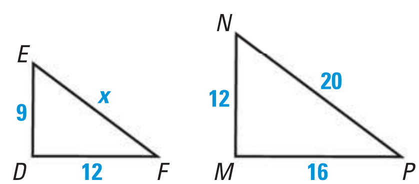
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

EXAMPLE 3 Use similar polygons

ALGEBRA **In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.**



Solution

The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$	**Write proportion.**
$\frac{12}{9} = \frac{20}{x}$	**Substitute.**
$12x = 180$	**Cross Products Property**
$x = 15$	**Solve for x.**

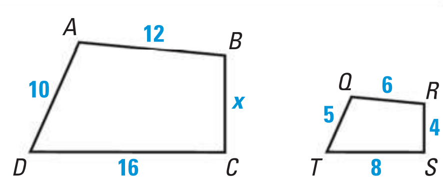
ANOTHER WAY
There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

✓ GUIDED PRACTICE for Examples 2 and 3

In the diagram, *ABCD* ~ *QRST*.

2. What is the scale factor of *QRST* to *ABCD*?

3. Find the value of x.

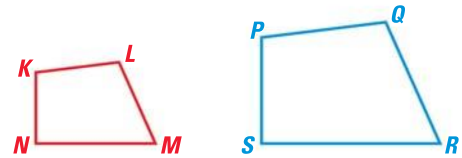


PERIMETERS The ratios of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

THEOREM — *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

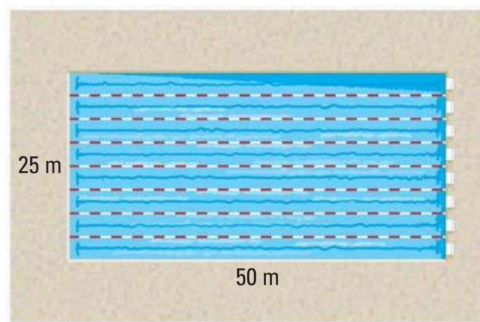


If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.



a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.

Solution

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

$\frac{x}{150} = \frac{4}{5}$ — Use Theorem 6.1 to write a proportion.

$x = 120$ — Multiply each side by 150 and simplify.

▶ The perimeter of the new pool is 120 meters.

ANOTHER WAY
Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:
$0.8(150) = 120$.

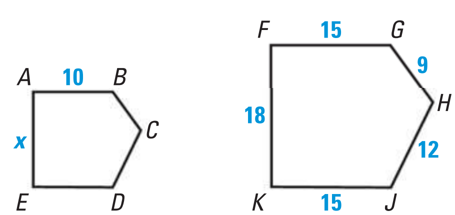
✓ GUIDED PRACTICE for Example 4

In the diagram, *ABCDE* ~ *FGHJK*.

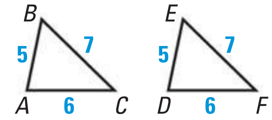
4. Find the scale factor of *FGHJK* to *ABCDE*.

5. Find the value of *x*.

6. Find the perimeter of *ABCDE*.



SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY

For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

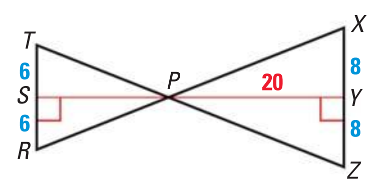
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6 + 6}{8 + 8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

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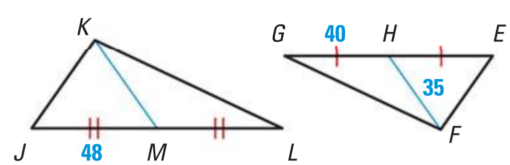
✓ GUIDED PRACTICE for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.



6.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 18, 27, 28, 35, 36, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

SKILL PRACTICE

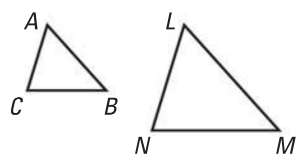
1. **VOCABULARY** Copy and complete: Two polygons are similar if corresponding angles are _?_ and corresponding side lengths are _?_.

2. ★ **WRITING** If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain*.

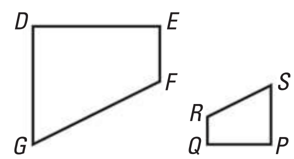
EXAMPLE 1 on p. 372 for Exs. 3–6

USING SIMILARITY List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.

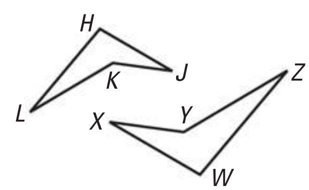
3. $\triangle ABC \sim \triangle LMN$



4. $DEFG \sim PQRS$



5. $HJKL \sim WXYZ$



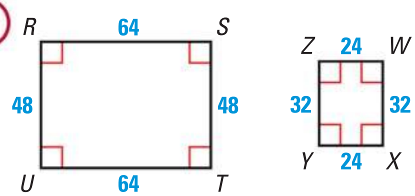
6. ★ **MULTIPLE CHOICE** Triangles *ABC* and *DEF* are similar. Which statement is *not* correct?

Ⓐ $\frac{BC}{EF} = \frac{BC}{EF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FD}$ Ⓒ $\frac{CA}{FD} = \frac{BC}{EF}$ Ⓓ $\frac{AB}{EF} = \frac{BC}{DE}$

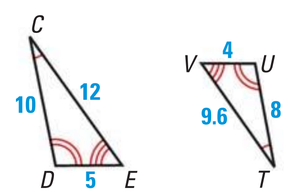
EXAMPLES 2 and 3 on p. 373 for Exs. 7–10

DETERMINING SIMILARITY Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

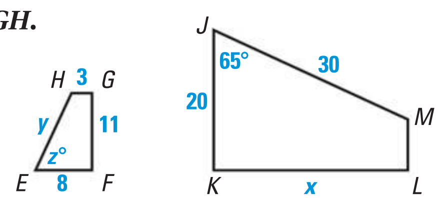
7.



8.



USING SIMILAR POLYGONS In the diagram, $JKLM \sim EFGH$.



9. Find the scale factor of *JKLM* to *EFGH*.

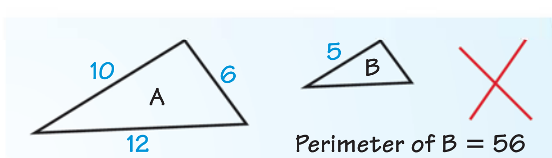
10. Find the values of *x*, *y*, and *z*.

EXAMPLE 4 on p. 374 for Exs. 11–13

11. Find the perimeter of each polygon.

12. **PERIMETER** Two similar FOR SALE signs have a scale factor of 5 : 3. The large sign's perimeter is 60 inches. Find the small sign's perimeter.

13. **ERROR ANALYSIS** The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B.



REASONING **Are the polygons *always*, *sometimes*, or *never* similar?**

14. Two isosceles triangles

15. Two equilateral triangles

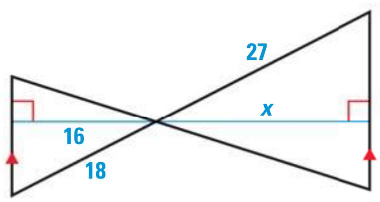
16. A right triangle and an isosceles triangle

17. A scalene triangle and an isosceles triangle

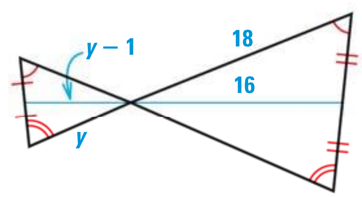
18. ★ **SHORT RESPONSE** The scale factor of Figure A to Figure B is $1:x$. What is the scale factor of Figure B to Figure A? *Explain* your reasoning.

SIMILAR TRIANGLES **Identify the type of special segment shown in blue, and find the value of the variable.**

19.



20.



EXAMPLE 5
on p. 375
for Exs. 21–22

USING SCALE FACTOR **Triangles *NPQ* and *RST* are similar. The side lengths of △*NPQ* are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of △*RST* is 8 inches long.**

21. Find the lengths of the other two sides of △*RST*.

22. Find the length of the corresponding altitude in △*RST*.

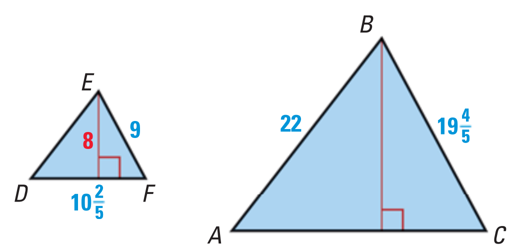
USING SIMILAR TRIANGLES **In the diagram, △*ABC* ~ △*DEF*.**

23. Find the scale factor of △*ABC* to △*DEF*.

24. Find the unknown side lengths in both triangles.

25. Find the length of the altitude shown in △*ABC*.

26. Find and compare the areas of both triangles.

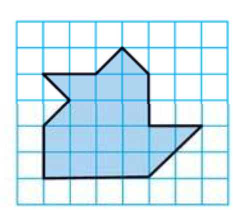


27. ★ **SHORT RESPONSE** Suppose you are told that △*PQR* ~ △*XYZ* and that the extended ratio of the angle measures in △*PQR* is $x : x + 30 : 3x$. Do you need to know anything about △*XYZ* to be able to write its extended ratio of angle measures? *Explain* your reasoning.

28. ★ **MULTIPLE CHOICE** The lengths of the legs of right triangle *ABC* are 3 feet and 4 feet. The shortest side of △*UVW* is 4.5 feet and △*UVW* ~ △*ABC*. How long is the hypotenuse of △*UVW*?

(A) 1.5 ft (B) 5 ft (C) 6 ft (D) 7.5 ft

29. **CHALLENGE** Copy the figure at the right and divide it into two similar figures.

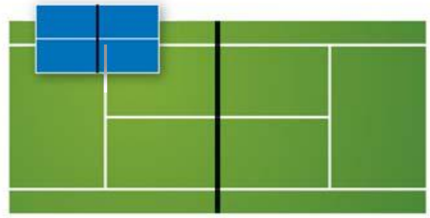


30. **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.

PROBLEM SOLVING

EXAMPLE 2 on p. 373 for Exs. 31–32

31. **TENNIS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain*. If so, find the scale factor of the tennis court to the table.



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32. **DIGITAL PROJECTOR** You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image.

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33. **MULTIPLE REPRESENTATIONS** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

a. **Making a Table** Copy and complete the table.

	AB	*BC*	*CD*	*DE*	*EA*
Figure 1	3.5	?	?	?	?
Figure 2	5.0	4.0	6.0	8.0	3.0

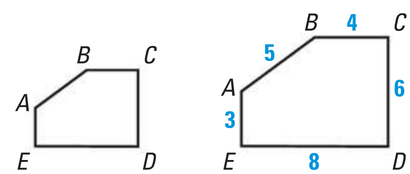
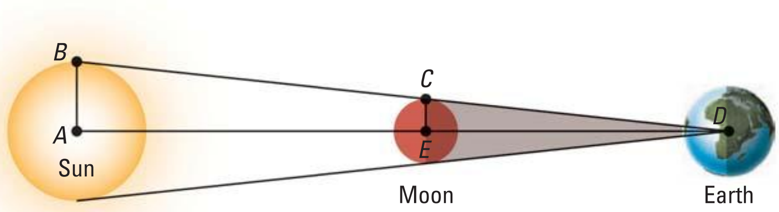


Figure 1 **Figure 2**

b. **Drawing a Graph** Graph the data in the table. Let x represent the length of a side in Figure 1 and let y represent the length of the corresponding side in Figure 2. Is the relationship linear?

c. **Writing an Equation** Write an equation that relates x and y. What is its slope? How is the slope related to the scale factor?

34. **MULTI-STEP PROBLEM** During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance *ED* between Earth and the moon is 240,000 miles, the distance *DA* between Earth and the sun is 93,000,000 miles, and the radius *AB* of the sun is 432,500 miles.



Not drawn to scale

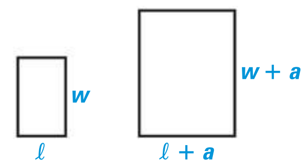
a. Copy the diagram and label the known distances.

b. In the diagram, $\triangle BDA \sim \triangle CDE$. Use this fact to explain a total eclipse of the sun.

c. Estimate the radius *CE* of the moon.

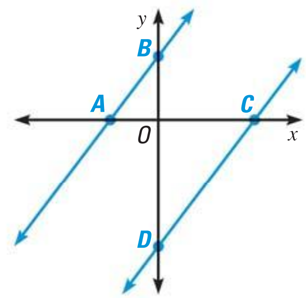
○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

35. ★ **SHORT RESPONSE** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.



36. ★ **SHORT RESPONSE** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

37. ★ **EXTENDED RESPONSE** The equations of two lines in the coordinate plane are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$.



a. *Explain* why the two lines are parallel.

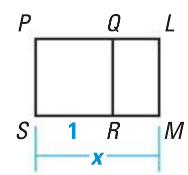
b. Show that $\angle BOA \cong \angle DOC$, $\angle OBA \cong \angle ODC$, and $\angle BAO \cong \angle DCO$.

c. Find the coordinates of points A, B, C, and D. Find the lengths of the sides of $\triangle AOB$ and $\triangle COD$.

d. Show that $\triangle AOB \sim \triangle COD$.

38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

39. **CHALLENGE** In the diagram, *PQRS* is a square, and $PLMS \sim LMRQ$. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



MIXED REVIEW

PREVIEW
Prepare for Lesson 6.4 in Exs. 40–42.

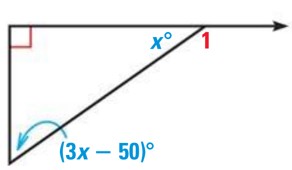
Given $A(1, 1)$, $B(3, 2)$, $C(2, 4)$, and $D\left(1, \frac{7}{2}\right)$, determine whether the following lines are *parallel*, *perpendicular*, or *neither*. *(p. 171)*

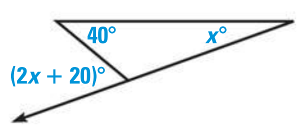
40. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

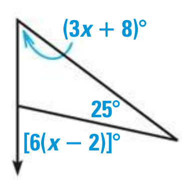
41. $\overleftrightarrow{CD}$ and $\overleftrightarrow{AD}$

42. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

Find the measure of the exterior angle shown. *(p. 217)*

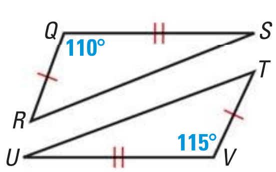
43.


44.


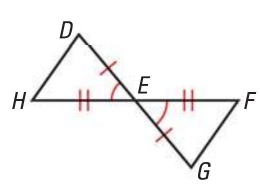
45.


Copy and complete the statement with <, >, or =. *(p. 335)*

46. RS __?__ TU



47. FG __?__ HD



48. WX __?__ YX